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Heat exchange by mass transfer in isothermal environment: A two-parameters non-linear model

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Abstract

Thermal engineering in buildings has lead to the development of standards to determine the performance of radiators. With the breakthrough of techniques such as cooling ceilings, a slightly different need has appeared: the parameters of characterization have to be independent of the size of the radiator. This study meets this new requirement by modifying the standard two-parameters power law. It is found that the new parameters are also independent of the mass flow that irrigates the panels. Experiments have been performed to confirm this result. We believe that this model is also attractive to other types of isothermal heat sink.

Keywords: Heat exchanger; Transfer; Radiant panel; Isothermal boundary condition; Standards; Non-linear characteristics

1. Introduction

Cooling buildings by use of hydraulic panels or irrigated slab (hydronic radiant systems) has recently seen an industrial growth. Practices in northern Europe have proved that hydronic cooled ceilings are able to remove high cooling loads without impairing thermal comfort. Compared to convective air conditioning systems, this technique provides several advantages: low indoor air velocity, fewer problems in maintaining permissible sound level, no risk of dry or sore throat and itching eyes [1,2]. In addition, it avoids any dissemination of infections as may happen when condensing units and ductwork are not cleaned regularly [2]. Moreover, it consumes less energy [3,4] and less space since water has a much higher heat capacity than air [1].

Despite these advantages, this technique has hardly been able to penetrate largely the market. This is partly because of fear of condensation on the ceiling [3]. Chilled radiators fitted with collectors have little cooling power because condensation limits the temperature difference between the air and the radiator surface [4]. A much better comfort is achieved by fitting large areas with hydronic panels. The total radiant surface must be large enough so that its temperature does not have to be decreased below dew point [5]. This strategy causes a pressure drop penalty because the flow should be turbulent along the entire radiant surface, a drawback that might be reduced with surfactant additives [6–8]. In addition to radiant cooling, air dehumidifying is usually still necessary [4]. Displacement ventilation is not appropriate in the case of a cooling ceiling, because only a small part of the heat load would be removed by the

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Nomenclature

A	area covered by the panels (m^2)	s	exponent of specific heat exchange
$\dot{c}$	specific heat capacity rate ($W\,K^{-1}\,m^{-2}$)	T	temperature (K)
c_p	heat capacity ($J\,kg^{-1}\,K^{-1}$)	T_{Ch}	temperature of the chamber walls (K)
g	efficiency performance index ($W\,K^{-1}\,m^{-2}$)	$T(x)$	mean temperature in a panel's cross-section, at coordinate x (K)
h	positive dimensionless number		
K	factor of heat exchange ($W\,K^{-n}$)		
k	factor of specific heat exchange ($W\,K^{-s}\,m^{-2}$)	*Greek symbols*	
k_{Log}	logarithmic factor of specific heat exchange ($W\,K^{-1}\,m^{-2}$)	$\theta(x)$	temperature difference
$\dot{M}$	mass flow ($kg\,s^{-1}$)	$\bar{\theta}$	average of $\theta(x)$ for all $x \in]0,1[$
$\dot{m}$	specific mass flow ($kg\,s^{-1}\,m^{-2}$)	$\bar{\theta}_{Log}$	logarithmic average of $\theta(x)$, see Eq. (11)
n	exponent of heat exchange	Δz	absolute error of approximation of a variable z
$\dot{Q}$	exchanged heat (W)	$O(z)$	residue operator: any real number whose absolute value is strictly below z
$\dot{Q}_{mass}(x)$	heat of mass transfer through the section of flow at coordinate x (W)	η	relative fall of temperature
r	intermediate parameter, $r = s - 1$; $0 \leqslant r < 1$	τ	intermediate parameter, see Eq. (48)

hydronic system [9]. An alternative consists of mounting a thermoelectric condenser upstream of the panels. The panel temperature can, by so doing, be kept above dew point by regulating the water flow [10–12].

Manufacturers are developing panels with increasing ability to exchange heat per unit of covered surface. A norm has recently appeared to standardize the test condition under which the steady state efficiency is measured (DIN4715 [13]). This standard imposes a two-parameters characterising method, similar to the empirical law used for characterizing room radiators (standard EN442 [14]). The obtained pair of parameters is independent of the temperature condition but may depend on the area of the panels and the mass flow. The present paper defines a new pair of characterizing parameters which do not depend on any of the above mentioned conditions. The mathematical model in which these parameters are defined is validated with experiments performed on two different types of panels.

The principle of our approach is to modify the well established European standard for characterizing heaters (EN442 [14]) in order to make its output independent of the radiating area. According to this standard, the heater is installed in a test chamber of $4\,m \times 4\,m \times 3\,m$, whose six walls are irrigated in order to set their interior surfaces to the same temperature (T_{Ch}, Fig. 1). The testing environment is then considered isothermal, even if there is some gradient of temperature in the chamber space.

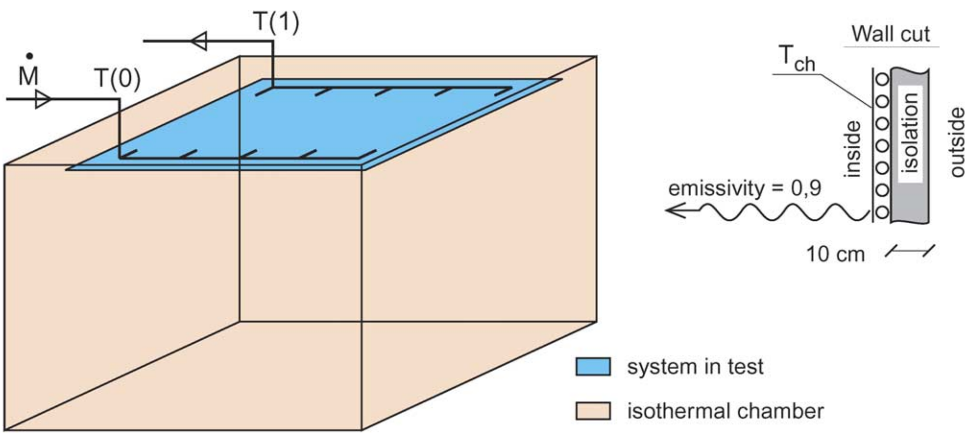

Fig. 1. Schema of the test chamber.

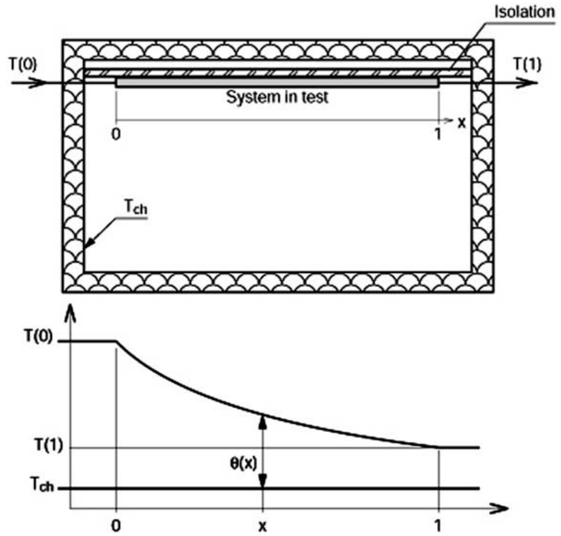

Fig. 2. Temperature difference between the system in test and the walls (in heating mode).

The panels in test bring heat into the room, or remove it, by means of a fluid. The set of panels in test is represented by a one-dimensional flow of equivalent heat transfer along a straight line of normalized coordinate x; $x = 0$ at the inlet and $x = 1$ at the outlet (Fig. 2). In steady state, the exchanged heat is expressed by the following equation (positive in heating mode):

$$\dot{Q} = c_p \cdot \dot{M} \cdot [T(0) - T(1)] \tag{1}$$

According to the EN 442 standard, this flux responds to the average difference of temperature between the panels and the room according to the following empirical law:

$$\dot{Q} = K \cdot (\bar{\theta})^n \tag{2}$$

Eq. (2) is adapted to both heating and cooling modes:

$$\dot{Q} = K \cdot |\bar{\theta}|^{(n-1)} \cdot \bar{\theta} \tag{3}$$

2. Basic statements

In order to introduce parameters that are independent of the area of panels, we consider an elementary panel section delimited by two planes, perpendicular to the x axis and at an infinitely small distance δx (Fig. 3). The entire panel is projected onto a surface of reference. In the present application, this is the (x, y) plane parallel to the wall on which the system is attached. The surface area resulting from the projection (denoted A in Fig. 3) forms a rectangle, whose width is also equal to A, since the x direction is normalized. In this rectangle, the elementary section delimits an area of $\delta A = A \cdot \delta x$. Besides, the averaged temperature in the section of coordinate x is considered in relation to the chamber walls temperature: $\theta(x) = T(x) - T_{\mathrm{Ch}}$ (Fig. 2). Applying the empirical law given by Eq. (3) to this elementary panel yields

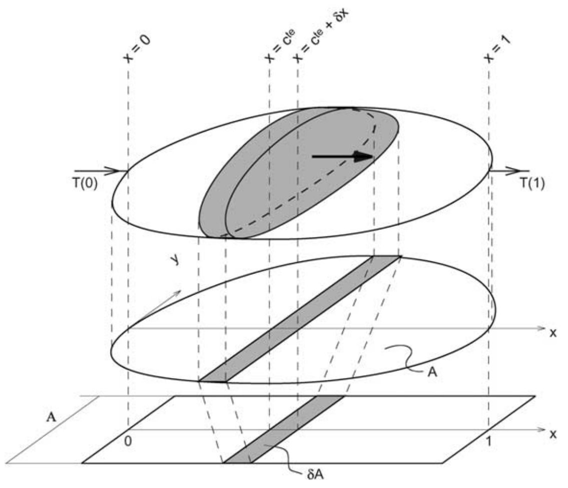

Fig. 3. An elementary volume that crosses the flow.

$$\forall x \in]0, 1[, \quad \delta\dot{Q}(x) = k \cdot A \cdot \delta x \cdot |\theta(x)|^r \cdot \theta(x) \tag{4}$$

The two parameters are now k and $s = 1 + r$, i.e. the factor and exponent of specific (per unit surface) heat exchange respectively. k and r are positive real numbers; r is maximum when the boundary layer is turbulent [15] and we define $0 \leqslant r < 1$.

The heat transferred by the mass flow $\dot{M}$ through an elementary section of the panel is

$$\forall x \in]0, 1[, \quad \delta\dot{Q}_{\mathrm{mass}}(x) = \dot{M} \cdot c_p \cdot \frac{\mathrm{d}\theta(x)}{\mathrm{d}x} \cdot \delta x \tag{5}$$

Taking a flow equivalent to the entire tested system, we do not consider any other heat flux. Energy balance between the panel and the chamber ($\delta\dot{Q}_{\mathrm{mass}}(x) + \delta\dot{Q}(x) = 0$) yields

$$\forall x \in]0, 1[, \quad \frac{\mathrm{d}\theta(x)}{\mathrm{d}x} = -\frac{k}{\dot{c}} \cdot |\theta(x)|^r \cdot \theta(x) \tag{6}$$

where $\dot{c}$ is the specific heat capacity rate which is the product of the heat capacity by the specific mass flow ($\dot{m} = \dot{M}/A$)

$$\dot{c} = c_p \cdot \dot{m} \tag{7}$$

Table 1 gives the typical values for water panels.

Table 1
Magnitude orders of specific heat capacity rate in water panels

$c_p/(\mathrm{J\ kg^{-1}\ K^{-1}})$	4×10^3
$\dot{m}/(\mathrm{kg\ s^{-1}\ m^{-2}})$	10^{-2}
$\dot{c}/(\mathrm{W\ K^{-1}\ m^{-2}})$	40

3. One-parameter model

For r set to zero, the parameter k is called logarithmic factor of specific heat exchange and denoted k_{Log}. An analytical solution has been derived by Antonopoulos [16] in the case of embedded tubes. The main results are recalled here for comparison with the general situation ($r \neq 0$). The temperature profile which has the value $\theta(0)$ at the inlet is

$$\theta(x) = \theta(0) \cdot \mathrm{e}^{-\frac{k_{\mathrm{Log}}}{\dot{c}} \cdot x} \tag{8}$$

The ratio $k_{\mathrm{Log}}/\dot{c}$ is equal to the Stanton number (see for example [17]).

Putting the outlet boundary condition $\theta(1)$ into Eq. (8), we obtain an expression that determines k_{Log} from the relative temperature decrease $\eta = \theta(1)/\theta(0)$, $0 < \eta < 1$

$$k_{\mathrm{Log}} = \dot{c} \cdot \mathrm{Ln}(1/\eta) \tag{9}$$

The average temperature along the flow $\bar{\theta} = \int_0^1 \theta(x) \cdot \mathrm{d}x$, $\bar{\theta}_{\mathrm{Log}}$ in the case of the one-parameter model, is deduced by integration of Eq. (8)

$$\frac{\bar{\theta}_{\mathrm{Log}}}{\theta(0)} = \frac{\dot{c}}{k_{\mathrm{Log}}} \cdot \left(1 - \mathrm{e}^{-\frac{k_{\mathrm{Log}}}{\dot{c}}}\right) \tag{10}$$

And, by applying Eq. (9)

$$\frac{\bar{\theta}_{\mathrm{Log}}}{\theta(0)} = \frac{1-\eta}{\mathrm{Ln}(1/\eta)} \tag{11}$$

Since we set $r = 0$, Eq. (4) becomes $\delta\dot{Q}(x) = A \cdot k_{\mathrm{Log}} \cdot \theta(x) \cdot \delta x$. Integration of this quantity along the flow gives all the exchanged heat

$$\dot{Q} = A \cdot k_{\mathrm{Log}} \cdot \bar{\theta}_{\mathrm{Log}} \tag{12}$$

This result compared to Eq. (2) shows that the one-parameter model leads to a linear characteristic law ($n = 1$), whose slope is the factor of heat exchange of the whole system

$$K = A \cdot k_{\mathrm{Log}} \tag{13}$$

But experiments reveal non-linear characteristics: in the case depicted in Fig. 5 for example, one can see that k_{Log} varies significantly. This justifies the introduction of the second parameter in the model.

4. Two-parameters model

4.1. Heat flux

We now solve the complete governing equation (Eq. (6)) for positive $\theta(x)$ only, since the negative case is symmetric. The solutions that have the initial value $\theta(0)$, are the following:

$$\theta(x) = \theta(0) \cdot (h \cdot x + 1)^{-1/r} \tag{14}$$

with $h = k \cdot r \cdot \theta(0)^r/\dot{c}$ a positive dimensionless number.

The relative temperature decrease $\eta = \theta(1)/\theta(0)$, applied in Eq. (14), yields

$$k = \frac{\dot{c}}{r \cdot \theta(0)^r} \cdot (\eta^{-r} - 1) \tag{15}$$

This equation provides a way to determine k if r is known. To determine r, one additional measured value at any $x \in]0, 1[$ is required.

The average temperature along the flow is calculated by integrating Eq. (14)

$$\frac{\bar{\theta}}{\theta(0)} = \frac{\dot{c}}{k \cdot \theta(0)^r} \cdot \frac{1-\eta^{1-r}}{1-r} \tag{16}$$

Besides, we have from Eq. (4) an expression of all the heat exchanged by the panels in test

$$\dot{Q} = k \cdot A \cdot \int_0^1 \theta(x)^{1+r} \cdot \mathrm{d}x \tag{17}$$

Integration of Eq. (14) power $1 + r$ yields

$$\dot{Q} = \dot{c} \cdot A \cdot \theta(0) \cdot [1 - (h+1)^{-1/r}] \tag{18}$$

From this equation, we observe that the exchanged heat depends linearly on the area (parameter A), if the specific heat capacity rate (parameter $\dot{c}$) is kept unchanged. This condition can be achieved by changing the mass flow proportionally to the area.

Furthermore, we note from Eq. (15) that

$$h + 1 = \eta^{-r} \tag{19}$$

As a check, we make the substitution in Eq. (18) and reobtain Eq. (1).

4.2. Useful approximation of r

As mentioned before, the parameter r could be extracted from one additional value of $\theta(x)$ at any x belonging to $]0, 1[$. But this is impossible in practice because we made the assumption of a one-dimensional flow. To measure r, the following method is introduced. We first extract the relative temperature decrease from Eq. (19)

$$\mathrm{Ln}(1/\eta) = (1/r) \cdot \mathrm{Ln}(h+1) \tag{20}$$

In the case of radiant panels, we note the following order of magnitude (Table 2):

$$h \ll 1 \tag{21}$$

Table 2
Typical values for radiant panels mounted on ceiling

$k/(\mathrm{W\,K}^{-(1+r)}\,\mathrm{m}^{-2})$	7
r	5×10^{-2}
$\theta(0)/\mathrm{K}$	10
$\dot{c}/(\mathrm{W\,K}^{-1}\,\mathrm{m}^{-2})$	40
$h = \frac{k \cdot r \cdot \theta(0)^r}{\dot{c}}$	1×10^{-2}
$h/r = \frac{k \cdot \theta(0)^r}{\dot{c}}$	0.2

Up to $h < 1/2$, we can approximate Eq. (20) by

$$\mathrm{Ln}(1/\eta) = \frac{k}{\dot{c}} \cdot \theta(0)^r \cdot [1 + \mathrm{O}(h)] \tag{22}$$

Taking two pairs of temperature difference values $(\theta(0)_1, \theta(1)_1)$ and $(\theta(0)_2, \theta(1)_2)$ at same specific heat capacity rate (parameter $\dot{c}$), with two different relative temperature falls η_1 and η_2, and with $h_1 < 1/2$ and $h_2 < 1/2$, we can write Eq. (22) in the form

$$k \cdot \theta(0)_1^r = -\dot{c} \cdot \mathrm{Ln}(\eta_1) \cdot [1 + \mathrm{O}(h_1)]^{-1};$$
$$k \cdot \theta(0)_2^r = -\dot{c} \cdot \mathrm{Ln}(\eta_2) \cdot [1 + \mathrm{O}(h_2)]^{-1} \tag{23}$$

The ratio of these two equations is

$$\left(\frac{\theta(0)_1}{\theta(0)_2}\right)^r = \frac{\mathrm{Ln}(\eta_1)}{\mathrm{Ln}(\eta_2)} \cdot \frac{1 + \mathrm{O}(h_2)}{1 + \mathrm{O}(h_1)} \tag{24}$$

The exponent of specific heat exchange can thus be determined by

$$s \cong 1 + \mathrm{Ln}\left(\frac{\mathrm{Ln}(\eta_1)}{\mathrm{Ln}(\eta_2)}\right) \Big/ \mathrm{Ln}\left(\frac{\theta(0)_1}{\theta(0)_2}\right) \tag{25}$$

The error made by approximating $(1 + \mathrm{O}(h_2))/(1 + \mathrm{O}(h_1))$ by unity is below $(h_1 + h_2)$; the absolute error of approximation is thus $\Delta s = |\mathrm{Ln}(1 + \mathrm{O}(h_1 + h_2))/\mathrm{Ln}(\theta(0)_1/\theta(0)_2)|$. Therefore

$$\Delta s < -\mathrm{Ln}(1 - (h_1 + h_2))/|\mathrm{Ln}(\theta(0)_1/\theta(0)_2)| \tag{26}$$

It appears that the more the temperature difference values $\theta(0)_1$ and $\theta(0)_2$ are apart, the better is the approximation. But h increases with $\theta(0)$. Therefore $\theta(0)$ should not be increased too much. For radiant panels (Table 2), at $\theta(0)_1 = 10$ K and $\theta(0)_2 = 2$ K, the boundary is below 2×10^{-2}.

4.3. Mean temperature of a panel

The mean temperature of panels in test, $\bar{\theta}$, is usually estimated using the logarithmic average, as in the standard DIN 4715 [13]. As shown in Section 3, this estimation corresponds to the one-parameter model. Eq. (16) gives the mean temperature of the two-parameters model. Making the ratio with Eq. (11), it follows:

$$\frac{\bar{\theta}}{\bar{\theta}_{\mathrm{Log}}} = \frac{k_{\mathrm{Log}}}{k \cdot \theta(0)^r} \cdot \left[1 - \frac{h}{(1+h)^{1/r} - 1}\right] \Big/ [1 - r] \tag{27}$$

On one hand, we know from Eq. (6) that the temperature profile $\theta(x)$ decreases more rapidly when r increases. We have thus necessarily

$$\bar{\theta}/\bar{\theta}_{\mathrm{Log}} < 1 \tag{28}$$

On the other hand, we observe that for $h < 1$

$$1 + h/r < (1 + h)^{1/r} \tag{29}$$

This yields $r > h/((1 + h)^{1/r} - 1)$ and thus

$$1 < \left[1 - \frac{h}{(1+h)^{1/r} - 1}\right] \Big/ [1 - r] \tag{30}$$

Besides, applying Eqs. (9), (15) and (20), the first term of the right side of Eq. (27) is simplified

$$\frac{k_{\mathrm{Log}}}{k \cdot \theta(0)^r} = \frac{\mathrm{Ln}(1+h)}{h} \tag{31}$$

Since $h > 0$

$$1 - \frac{h}{2} < \frac{k_{\mathrm{Log}}}{k \cdot \theta(0)^r} \tag{32}$$

With this result, plus Eqs. (28) and (30), one can see that Eq. (27) implies

$$1 - h/2 < \bar{\theta}/\bar{\theta}_{\mathrm{Log}} < 1 \tag{33}$$

Finally, h being small, the following approximation holds:

$$\bar{\theta} \cong \bar{\theta}_{\mathrm{Log}} \cdot (1 - h/4) \tag{34}$$

within the condition:

$$h < 1 \tag{35}$$

The relative error of the approximation Eq. (34) is bounded as follows:

$$\Delta\bar{\theta}/\bar{\theta}_{\mathrm{Log}} < h/4 \tag{36}$$

Considering radiant panels in normal condition of utilization (Table 2), this boundary is below 3‰.

4.4. Characteristic law

The characteristic law, Eq. (3), expresses the way the exchanged heat depends on the panels' mean temperature $\bar{\theta}$. As we have seen in the preceding section, the logarithmic average $\bar{\theta}_{\mathrm{Log}}$ can be used instead of $\bar{\theta}$; the introduced error stays bounded as shown by Eq. (36). In the case of a positive temperature profile ($\theta_{\mathrm{Log}} \geqslant 0$) and for $h < 1$, Eq. (3) is thus equivalent to:

$$\dot{Q} = K \cdot (\bar{\theta}_{\mathrm{Log}})^n \cdot (1 - [h + \mathrm{O}(h)]/4)^n \tag{37}$$

The two models must give the same exchanged heat. Equalling the right-hand sides of Eqs. (12) and (37) yields

$$K \cdot (\bar{\theta}_{\mathrm{Log}})^n \cdot (1 - [h + \mathrm{O}(h)]/4)^n = A \cdot k_{\mathrm{Log}} \cdot \bar{\theta}_{\mathrm{Log}} \tag{38}$$

Assuming

$$n = 1 + r \tag{39}$$

it follows:

$$K = A \cdot k_{\mathrm{Log}} \cdot (\bar{\theta}_{\mathrm{Log}})^{-r} \cdot (1 - [h + \mathrm{O}(h)]/4)^{-(1+r)} \tag{40}$$

Besides, since $h > 0$, Eq. (31) yields

$$k_{\mathrm{Log}} = k \cdot \theta(0)^r \cdot [1 - |\mathrm{O}(h)|/2] \tag{41}$$

And thus, within the conditions of Eq. (35), Eq. (40) implies that

$$K = A \cdot k \cdot \left(\frac{\theta(0)}{\bar{\theta}_{\text{Log}}}\right)^r \cdot \frac{1 - |\text{O}(h)|}{(1 - [h + \text{O}(h)]/4)^{1+r}} \tag{42}$$

The ratio $\theta(0)/\bar{\theta}_{\text{Log}}$ is obviously above 1, since $\bar{\theta}_{\text{Log}} < \theta(0)$. Searching for a boundary on the other side, we introduce Eq. (20) into Eq. (11). This yields

$$\frac{\theta(0)}{\bar{\theta}_{\text{Log}}} = \frac{1}{r} \cdot \frac{\text{Ln}(1+h)}{1 - (1+h)^{-1/r}} \tag{43}$$

Eq. (29) gives $1 - (1+h)^{-1/r} > (h/r) \cdot (1 + h/r)^{-1}$, and using $(1 + h/r)^{-1} > 1 - h/r$

$$1 - (1+h)^{-1/r} > (h/r) \cdot (1 - h/r) \tag{44}$$

In addition, making use of the property $\text{Ln}(1+h) < h$, it is found that Eq. (43) implies

$$\theta(0)/\bar{\theta}_{\text{Log}} < 1/(1 - h/r) \tag{45}$$

and, since within the condition $h/r < 1/2$ we can also use $1/(1 - h/r) < 1 + 2h/r$

$$\theta(0)/\bar{\theta}_{\text{Log}} < 1 + 2h/r \tag{46}$$

From Taylor series of the exponential around zero we know that $1 + 2h/r < \text{Exp}(2h/r)$. The term $(\theta(0)/\bar{\theta}_{\text{Log}})^r$ is thus bounded by Exp(2*h*). Besides, we know that if $1/4 > h$ then $\text{Exp}(2h) = 1 + 2h + \text{O}(4h^2)$. The condition $1/2 > h/r$ implies $r/2 > h$. So within the first condition $(h/r < 1/2)$ and $(r < 1/2)$, Eq. (43) yields:

$$(\theta(0)/\bar{\theta}_{\text{Log}})^r < 1 + 2h + 4h^2 \tag{47}$$

Finally, we obtain $\frac{1-|\text{O}(h)|}{(1-[h+\text{O}(h)]/4)^{1+r}} < \frac{K}{A \cdot k} < (1 + 2 \cdot h + 4 \cdot h^2) \cdot \frac{1-|\text{O}(h)|}{(1-[h+\text{O}(h)]/4)^{1+r}}$ by introducing Eq. (47) into Eq. (42), which leads to

$$1 - h < \frac{K}{A \cdot k} < \tau \text{ with } \tau = \frac{1 + 2h + 4h^2}{(1 - h/2)^{1+r}} \tag{48}$$

Note that τ is close to unity but greater and as h is small, we conclude that

$$K \cong A \cdot k \tag{49}$$

Besides, we also note that the upper limit of the error of approximation is larger than the lower one: $h < \tau - 1$. So we conclude, replacing the expression of h by its definition that, within the conditions

$$r < 1/2 \quad \text{and} \quad k \cdot \theta(0)^r / \dot{c} < 1/2 \tag{50}$$

the relative error is bounded as follows:

$$\frac{\Delta K}{A \cdot k} < \tau - 1 \tag{51}$$

For radiant panels in normal condition of utilization (Table 2), this boundary is below 3% ($r = 5 \times 10^{-2}$ and $k \cdot \theta(0)^r / \dot{c} = 0.2$).

The derivation made here shows that within the condition of Eq. (50), K is a pseudo-constant that depends only on the set of the panels in test. This results confirm our assumption made at Eq. (39) regarding the exponent parameters ($n \cong s$).

The fact that the well known empirical law Eq. (2) has been re-obtained by integration of Eq. (4) justifies "a posteriori" the invariance of the two parameters k and s with regard to the conditions of utilization (temperature, specific heat capacity rate and panel area). These parameters are thus inherently intrinsic and appropriate for characterization.

With the invariance of k and s one can explain the phenomena observed in the tests performed according to the standard EN442. For example, for a fixed temperature condition at the inlet of a given panel, the amount of exchanged heat depends on the mass flow, but one can observe that the measured points in the diagram $(\bar{\theta}, \dot{Q})$ stay on the same line, as long as the flow stays in normal range. Our two-parameters model allows to foretell this behaviour since it has shown that within the condition of Eq. (50), K and n are independent of the specific heat capacity rate.

5. Experimentation

The present characterisation technique is applied to hydraulic panels used in buildings. Mainly used in cooling mode, these panels are normally mounted onto the ceiling. In the present tests, the lower faces of the panels are situated at 2.74 m above the floor. Over 70% of the ceiling is covered (4 m × 4 m); the rest is filled with polystyrene slabs to avoid air circulation with the back volume. Two types of panels are tested, both insulated on the back side:

- AVE-PR from Energie Solaire SA: double sheet of painted stainless steel (see [18] for more details).
- CBA-CU12/60 from Barcol-Air SA: painted sheet of aluminium, tapped for acoustic softening and fitted with a hydraulic copper coil on the back.

For each temperature, the system is run till steady conditions are reached. Then the heat exchanged by the whole set of panels is measured using the relation Eq. (1). Plotting $\bar{\theta}$ and $\dot{Q}$ on a log-log diagram shows a linear distribution; the coefficient of correlation stays above 0.999 in all tested configurations (two types of panels, two mass flows, cooling and heating modes). This confirms the empirical law, Eq. (2), on which this study is based.

All the tests respect condition Eq. (50): the maximum of h is 2×10^{-2}. The measures are in good agreement with the three theoretical results established in the preceding section:

1. (K, n) is independent of the specific heat capacity rate: measures done at nominal mass flow (DN) and half of it (DN/2) do not show any significant change (Table 3).
2. The exponent of specific heat exchange s is determined with Eq. (25) by comparing measurements performed at equal mass flows. Table 4 gives the average and the standard-difference of the resulting set of data. In all investigated cases, the domain of incertitude includes the value of n found at the preceding stage (Table 3). Our experiments thus confirm the theory concerning the exponent parameter ($n \cong s$).
3. Knowing the value of s, the factor of specific heat exchange k can be determined by Eq. (15). As shown in Table 5, no significant differences have been detected between k and K/A. This confirms Eq. (49).

Table 3
Parameters of Eq. (3) for two values of mass flow—panel Energie Solaire

Mass flow	$K/(\mathrm{W\,K}^{-n})$		n	
	DN	DN/2	DN	DN/2
Cooling	78.8	78.9	1.07	1.07
Heating	67.3	68.3	1.05	1.04

Table 4
Exponent of specific heat exchange s; mean value (standard-difference)—panel Energie Solaire

Mass flow	DN	DN/2
Cooling	1.05 (0.03)	1.06 (0.02)
Heating	1.05 (0.02)	1.05 (0.01)

Table 5
Factor of specific heat exchange k, comparison with the ratio K/A—panel Energie Solaire

Mass flow	$K/A/(\mathrm{W\,K}^{-s}\,\mathrm{m}^{-2})$		$k/(\mathrm{W\,K}^{-s}\,\mathrm{m}^{-2})$
	DN	DN/2	
Cooling	6.8	6.8	7.0 ± 0.3
Heating	5.8	5.9	5.8 ± 0.1

Plotting the characteristic k versus inlet temperature (Fig. 4) confirms its high invariance. Plotting k_{Log} (Fig. 5), the equivalent of k in the one-parameter model (see Section 3), shows temperature dependence. A higher invariance is a definite advantage for a characterising parameter.

But on the other hand, two parameters are not appropriate for ranking. An index of efficiency is defined for that purpose. Its value in $\mathrm{W\,K^{-1}\,m^{-2}}$ is

$$g = k \cdot 10^{s} \tag{52}$$

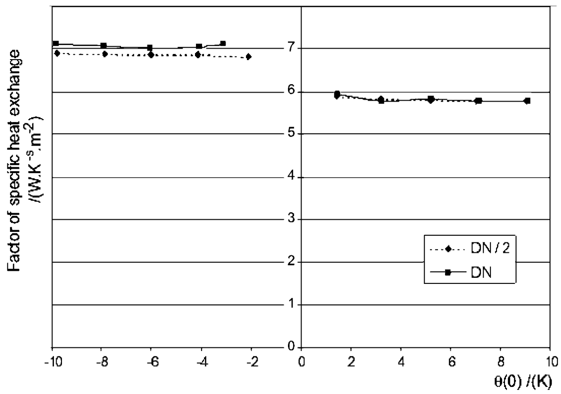

Fig. 4. Factor of specific heat exchange (k) of panel AVE-PR versus inlet temperature, at full and half nominal mass flow (DN and DN/2).

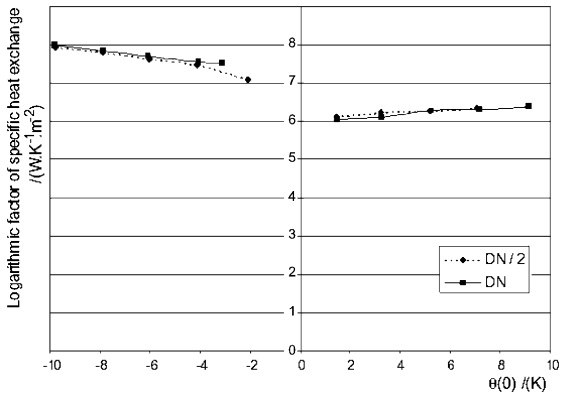

Fig. 5. Logarithmic factor of specific heat exchange (k_{Log}) of panel AVE-PR versus inlet temperature, at full and half nominal mass flow (DN and DN/2).

Table 6
Specific characteristics of the panel Barcol-Air and comparison with the one from Energie Solaire

	Barcol-Air CBA-CU12/60			Energie Solaire AVE-PR
	$k/(\mathrm{W\,K}^{-s}\,\mathrm{m}^{-2})$	s	$g/(\mathrm{W\,K}^{-1}\,\mathrm{m}^{-2})$	$g/(\mathrm{W\,K}^{-1}\,\mathrm{m}^{-2})$
Cooling	7.3 ± 0.2	1.06 ± 0.02	8.4 ± 0.3	7.9 ± 0.4
Heating	6.4 ± 0.1	1.06 ± 0.02	7.3 ± 0.2	6.5 ± 0.2

In Table 6 we give the characteristics of the second panels we tested: a higher efficiency is reached in cooling mode as well as in heating mode. By Eq. (41), the model foretells that g meets k_{Log} at $\theta(0) \cong 10$ K if h is small. Our measures confirm this (Fig. 5).

6. Conclusion

The present mathematical model of a heat exchanger is based on the empirical law used for characterizing heaters and radiators (standard EN442 [14]). It is a power law that depicts the exchanged heat versus the average temperature difference between the panels and the chamber, whose walls are set at a uniform temperature. The present model is derived by applying this law to an elementary piece of panel. Its two parameters, the factor and the exponent of heat exchange are thus specific characteristics, that is, per unit surface. An analytical expression of the total exchanged heat is derived. The resulting equation shows that the power of the system increases linearly with its area, if the specific heat capacity rate is kept unchanged.

From the expression of the total exchanged heat, the characteristic law of the complete heat exchanger is further derived. It is found that, under usual conditions (Eq. (50)), its parameters are nearly invariant with regard to the area of the panel and the specific heat capacity rate. Within these conditions, we find that the specific factor of heat exchange is simply equal to the standard factor divided by the area, and that the specific exponent is equal to the standard exponent. Experiments on two cooling ceilings confirmed these findings and the invariance of the characteristics with regard to temperature and mass flow. The temperature difference between the panels and the chamber is taken here relative to the interior wall temperature instead of using the air temperature, as prescribed by the standard EN442. The invariance feature regarding the specific heat capacity rate makes the present method cheaper by avoiding parametric studies. The invariance regarding area allows true comparison of panels of different sizes. These are also valuable advantages when comparing to the standard DIN4715 since the test chamber in this standard is not isothermal. In addition, this second standard is limited to cooling ceilings, whereas the method developed here allows characterizing all types of heat exchangers designed for indoor installation.

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